

HG(3) — Math (8) Prob.
Th. (Sc. & Arts)

2020

Time : 3 hours

Full Marks : 70

Pass Marks : 32

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any five questions.

1. (a) Give mathematical and statistical definitions of probability. What is difference between the two definitions of probability ?

(b) Two cards are drawn from a deck of well shuffled cards. What is the probability that the extracted cards are aces ?

2. (a) Define mutually exclusive events and compound events with examples. Prove multiplication theorem of probability.

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(Turn over)

(b) A bag contains 4 red and 3 blue balls. two drawing of 2 balls each are made. Find the chance that the first drawing gives 2 red balls

and the second drawing gives 2 blue balls :

(i) If the balls are returned to the bag after the first draw.

(ii) If the balls are not returned to the bag after the first draw.

3. (a) State and prove Baye's theorem.

(b) A problem in mechanics is given to three students A, B and C whose chance of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved ?

4. Define a random variable and its expectation. If X and Y are random variables, Prove that :

(i) $E(X + Y) = E(X) + E(Y)$

(ii) $E(X \cdot Y) = E(X) \cdot E(Y)$

Provided X and Y are independent random variables and E (X), E (Y) stand for their expectation.

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Contd.

5. Write Notes on any two of the following :

- (a) Kolmogrov's inequality
- (b) Convergence in probability
- (c) Almost sure convergence

6. Define variance of a random variable and prove any two of the following :

- (a) If the random variable is constant, its variance is zero
- (b) $\sigma^2 X = E X^2 - (E X)^2$
- (c) $\sigma^2(CX) = c^2 \sigma^2 X$

Where X is a random variable and C is a constant.

7. Define characteristic function. State and prove Uniqueness theorem of characteristic function.

8. Show that the probability of exactly j (j = 0, 1, 2, n) events denoted by S in

n Bernaullian trials is : $\frac{|n}{|j| |n-j|} p^j q^{n-j}$

Where p and q are the probabilities of the events S and F in a trial.

9. Prove Borel's strong law of large numbers.

10. (a) State and prove Bienayme's equality.

(b) A speaks the truth in 75 percent cases and B in 80 percent of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact ?

