2021

Time : 3 Hours

Maximum Marks : 70

D-267

Candidates are required to give their answers in their own words as far as practicable.

Answer any five questions

1. (a) Define metrizable and non-metrizable topological spaces with example for each of the two.

   (b) Prove that every discrete space is a metrizable space and an indiscrete space having at least two elements is not metrizable.

2. Let (x, τ) be a topological space and let $A \subseteq X$ and $x \in X$. If there exists a sequence $\{x_n\}$ of points of $A - \{x\}$ which converges to x, then prove that x is an accumulation point of A. Show that the converse does not hold in general.

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3. (a) Let (x, τ_1) and (Y, τ_2) be two topological spaces. Then prove that a function $f : X \to Y$ is $\tau_1 - \tau_2$ continuous if and only if the inverse image under f of every τ_2 – open set is τ_1 – open.

   (b) Define homomorphism on a topological space to another. Let (x, τ_1) and (Y, τ_2) be two topological spaces. Then prove that a one-one mapping f of x onto Y is a homomorphism if and only if $f(A^*) \subseteq [f(A)]^*$ for every sub set A of X.

4. (a) Prove that every compact sub set of a Hausdorff space is closed.

   (b) Prove that a set E in R (with its usual topology) is compact if and only if E is closed and bounded.

5. (a) Prove that a one-to-one continuous mapping of a compact topological space into a Hausdorff space is a homomorphism.

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(b) Examine, if every compact discrete space is finite.

6. (a) Define connectedness of a topological space. Let X be a topological space and A be a connected sub set of X. If B is a sub set of X such that $A \subseteq B \subseteq \overline{A}$, then prove that B is connected.

(b) Show that connectedness of a topological is not a hereditary property.

7. (a) Prove that a topological space X is disconnected if and only if there exists a continuous mapping of X onto the discrete two point space $Y=\{0,1\}$.

(b) If X and Y are topological spaces, then prove that the product space $X \times Y$ is connected if and only if X and Y are connected.

8. (a) Define T_0–spaces with example.

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Prove that every sub space of a T_0–space is a T_0–space.

(b) Prove that every compact Hausdorff space is a T_2-space

9. (a) Define first and second countable topological spaces and prove that every second countable space is also a first. Countable space but the converse is not necessarily true.

(b) Define Lindelof space. Prove that every second countable space is a Lindelof space.

10. (a) Show that a continuous image of a Lindelof space is a Lindelof space.

(b) Illustrate by giving a suitable example that Lindelofness is not a hereditary property.

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