HG(3) — Math (8)
Topology (Sc. & Arts)

2020

Time : 3 hours

Full Marks : 70

Pass Marks : 32

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any five questions.

1. (a) State Hausdorff's axiom system for topological space. Illustrate with a suitable example.

   (b) Prove that every metric space (X, d) is a metrizable space with the topology induced by the metric d.

2. Define convergence of a sequence in a topological space. Prove that in a Hausdorff 8.

http://www.lnmuonline.com

http://www.lnmuonline.com

http://www.lnmuonline.com

http://www.lnmuonline.com

space every convergent sequence has a unique limit. Also show that the condition for a topological space to be a Hausdorff space is not necessary for the uniqueness of limit of convergent sequences.

3. (a) Let (X, τ_1) and (Y, τ_2) be two topological spaces. Then prove that a function $f : X \to Y$ is $\tau_1 - \tau_2$ continuous if and only if the inverse image under f of every τ_2-closed sub-set of Y is a τ_1-closed subset of X.

   (b) Prove that a necessary and sufficient condition for a one to one onto mapping $f : X \to Y$, where X and Y are topological spaces to be homeomorphism is that $f(\bar{A}) = \overline{f(A)}$ for every $A \subset X$.

4. Define compact topological space. Prove that every closed subset of a compact space is compact but every compact subset of a topological space is not necessarily closed.

http://www.lnmuonline.com

http://www.lnmuonline.com

5. (a) Let (X, τ_1) and (Y, τ_2) be two topological spaces and let f be a continuous mapping of X into Y, then prove that for every compact subset E of X, f(E) is a compact subset of Y.

(b) Show that (R, U) is not a compact space where U is the usual topology on R.

6. (a) Let X be a topological space. If $\{A_i\}$ is a non-empty family of connected subsets of X such that $\bigcap A_i \neq \phi$, then prove $A = \bigcup A_i$ is also a connected subset of X.

(b) Construct a topological space which is compact but not connected.

7. (a) Prove that a continuous image of a connected set is also connected.

(b) Prove that a subspace X of the real line R with usual topology is connected if and only if X is an interval.

http://www.lnmuonline.com

http://www.lnmuonline.com

http://www.lnmuonline.com

(a) Define T_1-space. Prove that the property of being a T_1-space is both topological and hereditary.

(b) Prove that a topological space (X, τ) is a T_1-space if and only if every singleton subset {x} of X is a τ-closed set.

9. (a) Define T_2-space. Prove that every subspace of T_2-space is a T_2-sapce.

(b) Prove that every convergent sequence in a Hausdorff space has a unique limit.

10. Show that the following statements for a metrizable space X are equivalent :

(a) X is a second countable space

(b) X is a Lindelof space

(c) X is a separable space

http://www.lnmuonline.com

http://www.lnmuonline.com

http://www.lnmuonline.com