HG(3) — Math (8)
Num. Th. (Sc. & Arts)

2020

Time : 3 hours

Full Marks : 70

Pass Marks : 32

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any five questions.

1. (a) Prove that the total number of primes are infinite.

   (b) Find the solution of linear Diophantine equation $172x + 20y = 1000$.

2. (a) State and prove Fermat's little theorem.

   (b) If p is prime and p/ab then either p/a or p/b.

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3. (a) Define congruence and show that congruence is an equivalence ralation.

   (b) Find the remainder when 5^{48} is divided by 24.

4. (a) State and prove Wilson's theorem.

   (b) Show that $5^{38} \equiv 4$(mod II).

5. (a) Define multiplicative function. Let f(n) be a multiplicative function and let $F(n) = \sum_{d/n} f(d)$. Then show that F(n) is multiplicative.

   (b) Show that Mobius function is multiplicative.

6. (a) Define Legendre symbol. If p is a odd prime and a and b are any integers coprime to p, then prove that

$$\left(\frac{ab}{p}\right) = \left(\frac{a}{p}\right)\left(\frac{b}{p}\right).$$

(b) Show that every prime of the form $3k + 1$ is necessarily of the form $6m + 1$.

7. (a) State and prove Euler's criteria for quadratic residues.

(b) Solve the congruence $3x^2 + 5x + 9 \equiv 0 \pmod{11}$.

8. (a) What do you understand by partition of a number ? How can we represent a partition graphically ?

(b) Show that diophantine equation $x^4 + y^4 = z^2$ has no solution with non-zero positive integers x, y, z.

9. Show that every positive integer is the sum of four non-integral squares. Represent 333 as the sum of two integral squares

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10. Write short notes on any **two** of the following :

(a) Elementary properties of $\pi(x)$

(b) Elementary properties of $\mu(0)$

(c) Mobius Inversion Formula

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