HG(3) — Math (8)
Func. Anal. (New)
Sc. / Arts

2019

Time : 3 hours

Full Marks : 70

Pass Marks: 32

Candidates are required to give their answers in their own w0ords as far as practicable.

The questions are of equal value.

*Answer any **five** questions.*

1. (a) Define Banach space and givean example of normed space linear which is not a Banach space.

   (b) Show that the complex linear space C is Banach space under the norm $\| x \| = | x |$, $x \in C$.

2. (a) State and prove Holder's inequality.

(Turn over)

ST – 20/2

   (b) Prove that the Linear space C[0, 1] of real valued continuous functions on [0, 1] is a normed space.

3. (a) Define continuous linear functional on a normed linear space with an example.

   (b) Prove that a norm function is a continuous function.

4. (a) Let N and N′ be normed linear spaces and let T be a linear transformation of N into N′. Then show that T is continuous either at every point of N or at no point of N.

   (b) Let N and N′ be normed linear spaces and T be linear transformation of N onto N′ then prove that the null space of T is a Linear manifold and that Ker(T) is closed if T is continuous.

5. (a) State and prove Minkowski's inequality.

   (b) Prove that L^p- space is a normed linear space.

Contd.

6. State and prove Hahn-Banach theorem.

7. (a) State and prove Lemma of F-Reisz for normed linear space.

   (b) Construct a metric space which is not a normed linear space.

8. (a) State and prove polarisation identity in a Hilbert space.

   (b) State and prove parallelogram law in a Hilbert space.

9. (a) Construct a Banach space which is not a Hilbert space.

   (b) Prove that the inner product function is jointly continuous in a Hilbert space.

10. State and prove projection theorem in a Hilbert space.