HG(3) — M (8)
Func. Anal. / Sc.
& Arts – New

# 2018

*Time : 3 hours*

*Full Marks : 70*

*Pass Marks : 32*

*Candidates are required to give their answers in their own words as far as practicable.*

*The questions are of equal value.*

*Answer any five questions...*

1. (a) Define a normed linear space. In a normed linear space E, prove that $| \|x\| - \|y\| | \le \|x - y\|$ $\forall x, y \in E$.

   (b) Prove that the vector addition and scalar multiplicaction are continuous functions in the context of a normed linear space.

2. (a) Show that linear space $R^n$ is normed linear space under the norm $\|x\| = \left[\sum_{i=1}^{n}(x_i)^2\right]^{1/2}$.



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   (b) Show that the real linear spaces R is Banach spaces under the norm $\|x\| = |x|$, $\forall x \in R$.

3. (a) Let N be a normed linear space and $x_o$ a non-zero vector in N, then there exists a functional F in $N^*$ such that $F(x_n) = \|x_o\|$ and $\|F\| = 1$. Prove it.

   (b) Define Quotient space. Let M be a closed linear subspace in a normed linear space N. For each coset $x + M$ in the quotient space $N / M$ where $\|x + M\| = \inf\{\|x + M\| : m \in M\}$, then prove that $N / M$ is a normed linear space.

4. (a) Show that every normed linear space E is a metric space. With respect to metric defined as $d(x, y) = \|x - y\|$ $\forall x, y \in E$.

   (b) Show that the set C[a, b] of all continuous real functions defined on [a, b] is a real Banach space with respect to pointwise linear operations and norms defined by $\|f\| = \sup^{m} |f(x)|\, x \in (a, b)$.

5. Let T be a linear transformation of a normed linear space N into another linear space N′. Then prove that the following statement are equivalent :

   (a) T is continuous



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(b) $x_n \to 0 \Rightarrow T(x_n) \to 0$

(c) There exists a real number $K \geq 0$ such that $\| T(x) \| \leq K \| x \| \ \forall x \in N$.

(d) If $S = \{x : \| x \| \leq 1\}$ is closed unit sphere in N, then its image is a bounded set in N′.

6. (a) Let N and N′ be a normed linear space over the same scalar field and let T be linear transformation of N into N′ then show that T is bounded iff it is continuous.

(b) Prove that $l_p^n$ spaces are normed linear spaces, where $l_p^n$ of all n tuple $(x_1, x_2, \ldots x_n)$ and p be real number such that $1 \leq p < \infty$.

7. Let $x = \langle x_n \rangle$ and $y = \langle y_n \rangle$ be sequences or scalars (real or complex) and a be any scalar such that $\lim\limits_{n\to\infty} x_n = x_0$ and $\lim\limits_{n\to\infty} y_n = y_0$ then prove that :

(a) $\lim\limits_{n\to\infty} (x_n + y_n) = x_0 + y_0$

(b) $\lim\limits_{n\to\infty} a x_n = a x_0$







(c) lt [illegible] $- y_0$

(d) [illegible] $y_n \ \forall n$ [illegible] N [illegible] $y_0$

8. (a) Define inner product space and Hilbert space. In a Hilbert space, prove that :

(i) $(\alpha x - \beta y, z) = \alpha (x, z) - \beta (y, z)$

(ii) $(x, \beta y + \gamma z) = \bar{\beta} (x, y) + \bar{\gamma} (x, z)$

(b) Show that any inner product space can be embedded in a Hilbert space.

9. (a) Establish Schwartz's inequality in Hilbert space.

(b) Prove that in a Hilbert space the inner product space is jointly continuous.

10. (a) In a normed linear space prove that the closure of a convex set is convex.

(b) If x and y are any two vectors in a Hilbert space then show that :

(i) $\| x + y \|^2 - \| x - y \|^2 = 4 \operatorname{Re} (x, y)$

(ii) $(x, y) = \operatorname{Re} (x, y) + i \operatorname{Re} (x, iy)$

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