HG(3) — M (8)
Func. Anal. / Sc. &
Arts – New

2017

Time : 3 hours

Full Marks : 70

Pass Marks : 32

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer any **five** questions.*

1. (a) Define Banach space and give example of a normed linear space which is not a Banach space.

2015

   (b) Prove that norm function is a continuous function.

2. (a) State and prove Holder's inequality.

   (b) Show that in a normed linear space every convergent sequence is a Cauchy sequence.

(Turn over)

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3. State and prove Hahn-Banach theorem.

2015

4. (a) Define continuous Linear functional on a normed Linear space with a suitable example.

   (b) Construct a metric space which is not a normed Linear space.

5. (a) State and prove Minkowski's inequality.

   (b) Prove that the dual space of every normed Linear space is a Banach Space.

2015

6. (a) Prove that L^p-space is a normed Linear space.

   (b) Define inner product space and give an example of an incomplete inner product space.

7. (a) Prove that a closed sphere in a normed Linear space is a convex set.

   (b) Prove that every inner product space E is a normed linear space with respect to the

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norm defined by the inner product as $\|x\| = (x, x)^{\frac{1}{2}} = +\sqrt{(x, x)}$, for all $x \in E$ and where $<x, x>$ stands for the inner product of $x, y \in E$.

8. (a) Construct a Banach space which is not a Hilbert space.

(b) State and prove polarisation identity in a Hilbert space.

9. State and prove projection theorem in a Hilbert space.

10. (a) State and prove Parallelogram Law in a Hilbert space.

(b) Show that any inner product space can be embedded in a Hilbert space.

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