

HG (3) – Math (7) Sc & Arts

2021

Time : 3 Hours

Maximum Marks : 90

Candidates are required to give their answers in their own words as far as practicable.

Answer any **SIX** questions.

D-261

1. (a) Obtain the general conditions of equilibrium of a system of forces acting in one plane upon a rigid body.
- (b) Three forces P,Q,R act along the sides of the triangle formed by the lines $x + y = 1$, $y - x = 1$ and $y = z$. Find the equation to the line of action of the resultant.
2. (a) State and prove the principle of virtual work for any system of forces in one plane.

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- (b) A regular hexagon ABCDEF consist of six equal uniform rods, each of the weight w , freely joined together. The hexagon rest in a vertical plane and AB is in contact with a horizontal table. If C and F be connected by a light string. Prove that the tension is $w\sqrt{3}$.
3. (a) For a common catenary prove that $x = c \log(\sec \psi + \tan \psi)$
- (b) A telegraph wire stretched between two poles at a distance a metre apart, sags n metre in the middle. Prove that the tension at the end is approximately $w \left(\frac{a^2}{8n} + \frac{7}{6}n \right)$ where w is the weight per unit length.
4. Find the condition of stability for a body with one degree of freedom.
5. (a) Find the time period, amplitude and frequency in a S.H.M.
- (b) A particle starts with a given velocity V and moves under a retardation equal to K times the

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space described. Show that the distance traversed before it comes to rest is $\frac{v}{\sqrt{k}}$

(b) Show that $[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}] = [\vec{a} \quad \vec{b} \quad \vec{c}]^2$

6. (a) Prove that the work done against the tension in stretching a light elastic string is equal to the product of its extension and the mean of the initial and final tensions.
- (b) A mass hangs from a fixed point by a straight string and is given a small vertical displacement. Show that the motion is S.H.M.
7. Prove that the rate of change of momentum of a body in any given direction is equal to the resolved part of the external forces in the same direction.
8. (a) State and prove Kepler's law of central orbit.
- (b) A particle describes the circle $P^2 = ar$ under the force P to the pole. Find the law of forces.
9. (a) Prove that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c}) \cdot \vec{b} - (\vec{a} \cdot \vec{b}) \cdot \vec{c}$

10. (a) If \vec{a} and \vec{b} are differentiable vector functions of a scalar t , then prove that

$$\frac{d}{dt}(\vec{a} \times \vec{b}) = \vec{a} \times \frac{d\vec{b}}{dt} + \frac{d\vec{a}}{dt} \times \vec{b}$$

(b) Find the value of $\frac{d}{dt}[\vec{a}, \vec{b}, \vec{c}]$

11. (a) Prove that $\text{div}(\vec{a} \times \vec{b}) = \vec{b} \cdot (\text{Curl } \vec{a}) - \vec{a} \cdot (\text{Curl } \vec{b})$

(b) Prove that $(\vec{r} \cdot \nabla) \cdot \phi = \vec{r} \cdot (\nabla \phi)$

12. State and prove Stoke's theorem.

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