HG(3) — Math (7)
Sc. & Arts

2020

Time : 3 hours

Full Marks : 90

Pass Marks : 41

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer any **six** questions.*

1. (a) Prove that any system of forces, acting in one plane upon a rigid body can be reduced to either a single force or a single couple.

   (b) Two system of forces P, Q, R and P′, Q′, R′ act along the sides BC, CA, AB of a triangle ABC. Prove that their resultants will be parallel if $(QR' - Q'R)\sin A + (RP' - R'P)\sin B + (PQ' - P'Q)\sin C = 0$.

2. (a) Find the forces which may be omitted in forming the equation of virtual work.

   (b) Two equal uniform rods AB and AC, each of length 2b, are jointed at A and rest on a smooth vertical circle of radius a. Show that if 2θ be the angle between them then $b\sin^3\theta = a\cos\theta$.

3. (a) Derive the equation of common catenary in the form $y = c\cos h\, {}^{x}/_{c}$.

   (b) If α and β be the angles which a string of length ℓ makes with the vertical at the points of support, show that the height of one point above the other is $\dfrac{\ell\cos\frac{1}{2}(\alpha+\beta)}{\cos\frac{1}{2}(\beta-\alpha)}$

4. Establish the energy test for stability. A solid frustum of a paraboloid of revolution of height h and latus rectum 4a rests with its vertex on the vertex of a paraboloid of revolution, whose latus rectum is 4b show that the equilibrium is stable if $h < \dfrac{3ab}{a+b}$.

5. (a) Find the amplitude and frequency of the combined motion of two simple harmonic motions of the same period and in the same straight line.

   (b) A particle whose mass is m, is acted upon by a force $m\mu(x + \frac{a^4}{x^3})$ towards the origin O ; if it starts from rest at a distance a, show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$.

6. State Kepler's law of planetary motion and deduce the third law from Newton's law of gravitation.

7. (a) Find the differential of the central orbit in polar co-ordinates.

   (b) A particle describes the curve $v^n = a^n$. Cos $n\theta$ under a force P to the pole. Find the law of force.

8. (a) Show that if no external forces act on the system of particle moving on a straight line, the centre of inertia is either at rest or moves with uniform velocity.

   (b) Find the work done in extending a light elastic string to double its length.

9. (a) Define and interpret geometrically the scalar product of three vectors.

   (b) Prove :
   $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$

10. (a) Show that the necessary and sufficient condition for the vector function $\vec{V}$ of scalar variable t to have constant magnitude is $\vec{v} \cdot \frac{d\vec{v}}{dt} = 0$

    (b) If $\vec{a}$ is a unit vector, prove $\left|\vec{a} \times \frac{d\vec{a}}{dt}\right| = \left|\frac{d\vec{a}}{dt}\right|$

11. (a) Prove : Curl $(\phi \vec{a}) = \phi$ curl $\vec{a}$ + (grad . ϕ) $\times \vec{a}$

    (b) Prove : Div. (Curl $\vec{v}$) = 0

12. State and prove Green's theorem.