2019

Time : 3 hours

Full Marks : 90

Pass Marks : 41

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer any **six** quetsions.*

1. (a) Find the equation of the line of action of the resultant of a system of coplanar forces acting on a rigid body.

   (b) Forces P, Q, R act along the sides of the triangle formed by the lines $x = 0$, $y = 0$ and $x\cos\theta + y\sin\theta = p$, axes being rectangular. Find the magnitude of the resultant and equation of its line of action.

2. (a) State and prove the principles of virtual work for a system of coplanar forces.

   (b) The middle points of the opposite sides of a jointed quadrilateral are connected by light

rods of lengths ℓ and ℓ'. If T and T′ be the tensions of these rods, then prove that

$$\frac{T}{\ell}+\frac{T'}{\ell'}=0.$$

3. (a) Prove the following for a common catenary :

   (i) $y = c\sec\psi$

   (ii) $y^2 = s^2 + c^2$

   (b) Show that the length of a heavy endless chain which will hang over circular pulley of radius a so as to be in contract with two third of the circumference is :

$$a\left[\frac{3}{\log(2+\sqrt{3})}+\frac{4\pi}{3}\right]$$

4. Find the conditions of stability for a body with one degree of freedom.

5. (a) Define a simple harmonic motion. Prove that the time period of a S. H. M. is independent of its amplitude.

   (b) A body moves from rest from a point O so that its acceleration after t seconds from O is $\frac{1}{(t+2)^2}$. Find the distance discribed in 9 seconds and its velocity then.

6. (a) If V_1 and V_2 be the linear velocity of a planet when it is respectivley nearest and farthest from the sun, then prove that $(1-e)V_1 = (1+e)V_2$. https://www.lnmuonline.com

(b) Prove that the extension of a heavy elastic string of weight w and natural length ℓ hanging from one end and supporting a weight w′ at the is $\frac{\ell}{\lambda}(w' + \frac{1}{2}w)$, where λ is the modulus of elasticity of the string.

7. State D' Alembert's principle and prove that the rate of change of momentum of a body in any given direction is equal to the resolved part of the external forces in the same direction.

8. (a) Define minimum time of oscillator of a compound pendulum.

(b) Determine the motion of a rigid body acted on by the force of gravity only and moving about a fixed horizontal axis.

9. (a) Prove :

$$[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$$

(b) Prove :

$$b^2 \vec{a} = (\vec{a} \cdot \vec{b})\vec{b} + \vec{b} \times (\vec{a} \times \vec{b})$$

10. (a) If $\vec{v} \times \frac{d\vec{v}}{dt} = 0$, then show that $\vec{v}(t)$ is a constant vectors.

(b) Evaluate :

$$\frac{d}{dt}\left(r . \frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right)$$

11. (a) Define divergence and curl of a vector field. prove :

$$\text{div.}(\vec{a} \pm \vec{b}) = \text{div. } \vec{a} \pm \text{div. } \vec{b}$$

(b) Prove :

Curl (grad . ϕ) = 0

12. State and prove Stoke's theorem.

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