HG (3) – Math (6) Sc & Arts

2021

Time : 3 Hours

Maximum Marks : 90

Candidates are required to give their answers in their own words as far as practicable.

Answer any **SIX** questions.

D-260

1. Define inner automorphism.
Prove that the set I (G) of all inner automorphism of a group G is a normal sub group of the group of its automorphism and isomorphic to the quotient group $G/_{Z}$ of G, where Z is the centre of G.

2. (a) Define normalizer of an element of a group.
Prove that the normalizer $N(a)$ of $a \in G$ is a sub group of G.

(b) State and prove class equation of a finite group.

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3. (a) State and prove Cauchy's theorem for finite abelian group.

(b) If H is a p-sylow sub group of G and $x \in G$ then prove that $x^{-1}Hx$ is also a p-sylow sub group of G.

4. (a) Suppose R is a ring, S and ideal of R. Let f be a mapping from R to $^{R}/_{S}$ defined by $f(a) = S + a$, $\forall a \in R$. Then prove that f is a homomorphism of R onto $^{R}/_{S}$.

(b) Prove that an ideal S of a commutative ring R with unity is maximal if and only if the residue class ring $^{R}/_{S}$ is a field.

5. (a) Define Euclidean Ring.
Prove that the ring of polynomials our a field is a Euclidean ring.

(b) Prove that every Euclidean ring is a principal ideal ring.

6. Define unique Factorization Domain.
Prove that every Euclidean Domain is a unique Factorization Domain.

7. Define vector space.
Let V(F) be a vector space and 0 be the zero vector of V then prove :
 (a) $a.0 = 0, \forall a \in F$
 (b) $0.\alpha = 0, \forall \alpha \in V$
and (c) $a.\alpha = 0 \Rightarrow a = 0$ or $\alpha = 0$

8. (a) If W_1 and W_2 are sub spaces of a vector space V(F) then prove :
 (i) $W_1 + W_2$ is a sub space of V(F)
and (ii) $L(W_1 \cup W_2) = W_1 + W_2$
 (b) Show that the vectors (1,2,1), (2,1,0), (1,-1,2) form a basis of R^3

9. Prove that the set S of all linear transformations from a vector space V(K) into a vector space U(K) is a vector space over the field F relative to the operations of vector addition and scalar multiplication defined as:

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$$(T_1 + T_2)(x) = T_1(x) + T_1(x)$$
and $(a\, T_1)(x) = +a\, T_1(x), \forall x \in V, a \in k$
and $T_1, T_2 \in S$

10. (a) If A is a non-singular matrix. Then show that the eigen values of A^{-1} are the reciprocals of the eigen values of A and conversely.
 (b) Find the eigen values and eigen vectors of the matrix
$$A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$$

11. (a) Introduce the concept of an inner-product space and prove that every inner product space is a normed linear space but not conversely.
 (b) Construct a Banach space which is not a Hilbert space.

12. (a) Introduce the concepts of module and sub-module with examples illustrating them.
 (b) Prove that every abelian group G is a module over the ring of integers.

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