HG (3) - Math (5) Sc. & Arts

2021

Time : 3 Hours

Maximum Marks : 90

D-259

Candidates are required to give their answers in their own words as far as practicable.

Answer any six questions.

1. If f be a Bounded function on the bounded interval $[a,b]$. Then show that $f \in R[a,b]$ if and only if, for every $\epsilon > 0$, there exists a partition P of $[a, b]$, such that $U(P, f) - L(P, f) < \epsilon$.

2. (a) If f is monotonic on $[a, b]$ then show that f is R-integrable on $[a, b]$.

   (b) If $f \in R[a,b]$ then $|f| \in R[a,b]$, and

$$\left|\int_a^b f\right| \leq \int_a^b |f|$$

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3. (a) State and prove Abel's test for the convergence of the integral of a product of two functions.

   (b) State comparison test for the convergence of an improper integral and hence test the convergence of the integral

$$\int_0^\infty \frac{Cosx}{1+x^2}dx$$

4. (a) State and prove young's theorem,

   (b) Examine the continuity and different ability of the function

$$f(x,y) = \frac{xy^2}{x^2+y^2}, (x,y) \neq (0,0)$$

$$f(0,0) = 0 \ at \ (0,0)$$

5. (a) Define analytic function. Find the necessary and sufficient condition for f (z) to be analytic

   (b) Show that the function $f(z) = xy + iy$ is every where continuous but not analytic.

6. (a) Define bilinear transformation. Show that the resultant of two Bilinear transformation is a bilinear transformation.

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(b) What is cross ratio? Show that the cross-ratio of four points is invariant under a bilinear Transformation.

7. (a) Define harmonic functions. Show that if $f(z) = u + iv$ is an analytic function, than u and v both are harmonic functions.

(b) Show that the function $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic. Also find its harmonic conjugate.

8. (a) Define inverse points. Show that the inverse of a point a with respect to the circle $|z - c| = r$ is the point $c + \frac{r^2}{\bar{a} - \bar{c}}$

(b) Obtain the condition for four points to be Concyclic.

9. (a) Let M be a non-empty set. Then a mapping d of M×M into R is a metric on M iff

(i) $d(x, y = o \Leftrightarrow x = y\ for\ all\ x, y \in M$

(ii) $d(x, z) \leq d(x, y) + d(z, y) for\ all\ x, y, z \in M$

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(b) Let (E, e) be a metric space. Then (E, d) is a metric space where d is defined by $d(x, y) = \frac{e(x,y)}{1+e(x,y)}$ for all, x, y of E.

10. (a) Let (x, d) be a metric space, then prove that every closed sphere is X is a closed set relative to the d-metric topology for X.

(b) In a metric space (x, d) prove that the intersection of two open sets is open

11. (a) State and prove cantor's Intersection theorem

(b) Show that any contraction map T on a metric space (E, d) is uniformly continuous.

12. Prove that every compact metric space is complete.

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