HG(3) — Math (5) New
(Sc. / Arts)

2019

Time : 3 hours

Full Marks : 90

Pass Marks : 41

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any ***six*** *questions.*

1. (a) Eatablish the equivalence of bound definition and limit definition of Riemann-Integration.

   (b) Show that the function f(x) defined in the internal [0, 1] such that :

   $f(x) = \frac{1}{2^n}$ where $\frac{1}{2^{n+1}} < x \leq \frac{1}{2^n}$

   $f(0) = 0$

   where n = 0, 1, 2, 3 ·········· is integrable over [0, 1] and evaluate $\int_0^1 f(x)dx$.

2. (a) If f is monotonic on [a, b], then show that f is R-integrable on [a. b].

   (b) State and prove 1st mean value theorem.

3. (a) State and prove Abel's test for the convergence of the integral of a product of two functions.

   (b) Discuss the convergence of $\int_0^{\pi/2} \log \sin x \, dx$.

4. (a) State and prove Schwarrtz's theorem.

   (b) Show that $f(x, y) = xy \frac{x^2 - y^2}{x^2 + y^2}$ if $x^2 + y^2 \neq 0$

   $= 0$ if $(x, y) = (0, 0)$

   is differentiable at the origin.

5. (a) Define analytic function. Find the necessary and sufficient condition for f(z) to be analytic.

   (b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied at that point.

6. (a) Define bilinear transformation. Prove that the resultant of two bilinear transformation is a linear transformation.

(b) Prove that the cross-ratio of four points is invariant under a bilinear transformation.

7. (a) Define continuity and differentiability of a complex function f(z) in a domain D. Prove that differentiability implies continuity.

(b) Prove that the function $f(z) = |z|^2$ is continuous everywhere but is nowhere differentiable except at the origin.

8. (a) Define inverse points. Show that the inverse of a point a with respect to the circle $|z - c| = r$ is the point $c + \frac{r^2}{\bar{a} - \bar{c}}$.

(b) Obtain the condition for four points to be concyclic.

9. (a) Let X be of a non-empty set and d be a real valued function of X × X into R. Then prove that d is a metric iff :

(i) $d(x, y) = 0 \Leftrightarrow x = y$

(ii) $d(x, y) \leq d(x, z) + d(y, z)$

(b) Let (X, d) be a metric space and $d^*(x, y) = \min\{1, d(x, y)\}$. Prove that d^* is a metric for x.

10. (a) Let (X, d) be a metric space, then prove that every closed sphere in X is a closed set relative to the d-metric topology for X.

(b) In a metric space (X, d) prove that the intersection of two open sets is open.

11. (a) Prove that every metric space is a Hausdorf space.

(b) State and prove Baire's Category theorem.

12. Prove that every compact metric space is complete.

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