# 2021

Time : 3 Hours

Maximum Marks : 70

Candidates are required to give their answers in their own words as far as practicable.

D-268

Answer any five questions

1. (a) Define normed linear space. Give an example of a metric space which is not a normed linear space.

   (b) Show that the real linear space R and the complex linear space C are Banach spaces under the norm :

$$\|x\| = |x|, \qquad x \in C \text{ or } R.$$



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2. (a) State and prove Holder's irregularity.

   (b) Let N be a normed linear space. Then prove that the mapping $f: N \to R$ such that $f(x) = \|x\|$ is continuous.

3. (a) Prove that $L_p$ spaces are normed linear spaces.

   (b) Prove that a non-zero normed linear space $N$ is a Banach space if and only if $S = \{x: \|x\| = 1\}$is complete.

4. Let $\langle x_n \rangle$ and $\langle y_n \rangle$ be two sequences of scalars such that $x_n \to x_0$ and $y_n \to y_0$ as $n \to \infty$ and $\propto$ by any scalar. Then prove :

   (a) $\underset{n\to\infty}{lin}(x_n + y_n) = x_0 + y_0$

   (b) $\underset{n\to\infty}{Lt} \propto x_n = \propto x_0$

   (c) $\underset{n\to\infty}{lin} (x_n - y_n) = x_0 - y_0$

   (d) If $x_n < y_n, \forall_n$ then $x_0 < y_0$



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5. Prove that dual space of every normed linear space is a Banach space.

6. (a) Let M be a closed linear subspace of a normed linear space N and let $\phi$ be a natural mapping defined by $\phi(x) = x + M$. Then prove that $\phi$ is a continuous linear transformation for which $\|\phi\| \leq 1$.

(b) Let N and $N^1$ be normed linear operators and $T: N \to N^1$ be a linear transformation. Then prove that Ker. (T) is a linear manifold and that Ker (T) is closed if T is continuous.

7. State and prove Hahn – Banach theorem.

8. (a) Define inner product space and Hilbert space. Prove that in a Hilbert space :

(i) $(\propto x - \beta y, Z) = \propto (x, z) - \beta(y, z)$

(ii) $(x, \beta y + rZ) = \bar{\beta}(x, y) + \bar{r}(x, z)$





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(b) Prove that inner product is jointly continuous.

9. (a) State and prove Parallelogram law in Hilbert space.

(b) Let H be a Hilbert space and $x, y$ be only two vectors of H, then prove :

(i) $\|x + y\|^2 - \|x - y\|^2 = 4\, Re\, (x, y)$ and

(ii) $(x, y) = Re\,(x, y) + i\, Re\,(x, iy)$

10. State and prove Bessel's inequality in a Hilbert space.

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