HG(3) — Math (8)
Func. Anal.
(Sc. & Arts)

2020

Time : 3 hours

Full Marks : 70

Pass Marks : 32

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer any **five** questions.*

1. (a) Define Normed Linear space. Let N be a normed linear space and let d be the function from N × N into R defined by $d(x, y) = \| x - y \|$. Then prove that d is a metric on N.

(b) Prove that in a normed linear space, every convergent sequence is a Cauchy Sequence.

2. (a) Define Banach space. Prove that a normed linear space N is a Banach space if and only if every absolutely summable series in N is summable.

(b) Let N be a normed linear space and let x_0 y ∈ N. Prove :

$$\left| \| x \| - \| y \| \right| \le \| x - y \|$$

3. (a) State and prove Minkowski's inequality.

(b) Show that the linear space R^n of all n-tuples $x = (x_1, x_2, \ldots\ldots\ldots x_n)$ of real numbers are Banach space under the norm $\| x \| = \left(\sum_{i=1}^{n} |x_i|^2 \right)^{1/2}$.

4. (a) Let N be a normed linear space and let F denotes C or R then the mapping

$$f : N \times N \to N : f(x, y) = x + y$$

and $g : F \times N \to N : g(\alpha, x) = \alpha . x$ are continuous.

(b) Let C(X) denote the linear space of all bounded continuous scalar-valued functions defined on a topological space X. Show that C(X) is a Banach space under the norm $\| f \| = \sup. \{ 1f(x) | : x \in X\}$.

5. Define Quotient spae. Let M be a closed linear sub space in a normed linear space N. For each coset x + M in the quotient space N/M we define $||x + M|| = \inf. \{||x + m|| : m \in M\}$.

Then prove N/M is a normed linear spaces. Also prove that if N is a Banach space, the N/M is also a Banach space.

6. Let N and N′ be normed linear spaces and B(N, N′) denote the set of all bounded or continuous linear transformations from N to N′. Then prove that B(N, N′) is itself a normed linear space with respect to pointwise linear operations :

$$(T + U)(x) = T(x) + U(x)$$

$$(\alpha T)x = \alpha T(x)$$

and the norm is defined by

$$||T|| = \sup. \{||T(x)|| : x \in N, ||x|| < 1\}.$$

Further prove that if N′ is a Banach space then so is B(N, N′)

7. (a) Let N be a normed linear space and x_0 is a non-zero vector in N, then prove that

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there exists a functional F in N^*. Such that $F(x_0) = ||x_0||$ and $||F|| = 1$. where N^* is the conjugate space of N.

(b) Prove that a normed linear space is separable if its conjugate space is separable.

8. (a) State and prove Schwartz inequality.

(b) If L is an inner product space then show that $\sqrt{(x, x)}$ has the properties of a norm.

9. If B is a complex Banach space, whose norm obeys the parallelogram law and if an inner product is defined on B by $4(x, y) = ||x + y||^2 - ||x - y||^2 + i||x + iy||^2 - i||x - iy||^2$, then show that B is a Hillbert space.

10. (a) State and prove Pythagorean theorem.

(b) Let S be a non-empty sub set of a Hilbert space H. Then prove that $S^{\perp}$ is a closed linear subspace of H.

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