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2019

Time : 3 hours

Full Marks : 100

Pass Marks : 33

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer **eight** questions, selecting at least **one** from each Group.*

Group – A

1. (a) State and prove Leibnitz's theorem to find the n^{th} derivative of a product of two functions.

   (b) If $u = \tan^{-1}\dfrac{x^3+y^3}{x-y}$, show that

   $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 24.$$

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2. (a) State and prove Maclaurin's theorem for expansion of a function f(x) in terms of ascending powers of x.

   (b) Evaluate :

   $$\lim_{x\to 0}(\cos x)^{\cot^2 x}$$

3. (a) Show that in the exponential curve $y = be^{x/a}$, the subtangent is of constant length and subnormal series as the square of the ordinate.

   (b) Find the radius of curvature in cartesian form.

4. (a) Show that :

   $$\lim_{x\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots..+\frac{1}{2n}\right]=\frac{\pi}{4}$$

   (b) If $I_n = \int \tan^n x\,dx$, show that :

   $$I_n = \frac{\tan^{n-1}x}{n-1} - I_{n-2}.$$

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5. Find the area of the cardioide $r = a(1 - \cos\theta)$. Also find its perimeter.

6. (a) If $\overline{|(n)} = \int_0^\infty x^{n-1} e^{-x}\, dx$ show that :

$\overline{|(n)} = (n-1)(n-2) \cdots\cdots 3.2.1 \overline{|(1)}$ and $\overline{|(1)} = 1$.

(b) Evaluate :

$\iiint_R u^2 v^2 w \, du\, dv\, dw$

where R is the region $u^2 + v^2 \le 1$, $0 \le w \le 1$.

7. Solve any **two** of the following differential equations :

(a) $\frac{dy}{dx} = e^{x+y} + x^2 e^y$

(b) $\left(x^2 - y^2\right)\frac{dy}{dx} = 2xy$

(c) $\frac{dy}{dx} = \frac{x + 2y - 3}{2x + 4y - 3}$

(d) $\frac{dy}{dx} + 2y \tan x = \sin x$

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8. (a) Solve any **one** of the following :

(i) $y + px = x^4 p^2$

(ii) $y = px + \sin^{-1} p$

(b) Find the orthogonal trajectories of $r\theta = a$.

9. Solve any **two** of the following differential equations :

(a) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{3x}$

(b) $\frac{d^2y}{dx^2} + 4y = \sin 3x + e^x + x^2$

(c) $\frac{d^2y}{dx^2} - y = x(1 + x)e^{2x}$

Group – B

10. (a) Define scalar product of three vectors and show that in the scalar triple product, the dot and cross can be interchanged without changing the value of the result.

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(b) Prove that :

$$\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right]=2\left[\vec{a}, \vec{b}, \vec{c}\right]$$

11. (a) Show that the necessary and sufficient condition for the vector function $\vec{a}(t)$ to have constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt}=0$.

(b) If $\vec{a}$, $\vec{b}$ and w are constant and $\vec{r}=\vec{a}\cos wt + \vec{b}\sin wt$, then prove that $\frac{d^2\vec{r}}{dt^2}+w^2\vec{r}=0$.

12. (a) If θ and $\vec{A}$ are continuously differentiable scalar and vector functions respectively, then prove that :

$$\text{div}(\phi\vec{A})=\phi\,\text{div}\vec{A}+(\text{grad}\phi)\cdot\vec{A}$$

(b) Prove that curl(grad ϕ) = 0.

Group – C

13. (a) Obtain the general condition of equilibrium of a system of forces acting in one plane upon a rigid body.

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(b) The forces P, Q, R act along the sides BC, AC, BA of an equilateral triangle ABC. If their resultant is a force parallel to BC through the centroid of the triangle prove that $Q=R=\frac{1}{2}P$.

14. (a) State and prove the principle of virtual work for any system of forces in one plane.

(b) Enumerate the nature of forces which may be omitted in forming the equation of virtual work, giving reasons why they may be omitted.

15. A particle moves in a straight line OA starting from rest at A and moving with an acceleration which is always directed towards O and varies as the distance from O, discuss the motion.

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16. Obtain the tangential and normal components of velocity and acceleration of a particle moving along a curve in a plane.

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