2021

Time : 3 Hours

Maximum Marks : 90

D-812

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer any six questions.

1. (a) Prove the following in a ring ?

(i) $a(-b) = -(ab) = (-a)b$

(ii) $a(b-c) = ab - ac$

(iii) $(-a)(-b) = ab$

(b) Show that every field is an integral domain.

2. Solve any two of the following :

(i) $\frac{dy}{dx} = (x+y)^2$

(ii) $(x^2 - y^2)\frac{dy}{dx} = 2xy$

(iii) $x\frac{dy}{dx} + y = y^2 \log x$

(iv) $x\,dx + y\,dy + \frac{x\,dy - y\,dx}{x^2+y^2} = 0$

3. Solve any two of the following :

(i) $p^2 + 2py\cot x = y^2$

(ii) $y = apx + bp^2,\ p = \frac{dy}{dx}$

(iii) $y = xp + \frac{a}{p}, p = \frac{dy}{dx}$

(iv) Putting $x^2 = u$ and $y^2 = v$ reduce the equation $x^2 (y - xp) = yp^2$ into Clairaut's form and hence solve it, where $p = \frac{dy}{dx}$

4. Solve any two of the following differential equations :

(i) $\frac{d^2y}{dx^2} + y = \sin 2x$

(ii) $\frac{d^2y}{dx^2} + 4y = x^3 + e^x + \sin x$

(iii) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$

(iv) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x \cos x$

5. (a) Find the orthogonal trajectories of the family of cardioid $r = a(1 + \cos\theta)$

(b) Using method of variation of parameters solve the following differential equation

$$(x + 2)\frac{d^2y}{dx^2} - (2x + 5)\frac{dy}{dx} + 2y = (1 + x)e^x$$

6. Solve any two of the following differential equations :

(i) $(mz - ny)p + (nx - lz)q = ly - mx$

(ii) $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$

(iii) $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

(iv) $(y^2 + z^2 - x^2)p - 2xyq = -2xz$

where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$

7. Prove the following recurrence relations for the Legendre's polynomial Pn (x) :

(i) $nP_n(x) = (2n-1)x\,P_{n-1}(x) - (n-1)$

$P_{n-2}(x)$

(ii) $nP_n(x) = n\,P_n'(x) - P_{n-1}'(x)$

8. Prove the following relations for $J_n(x)$:

(i) $2n\,J_n(x) = x\{J_{n-1}(x) + J_{n+1}(x)\}$

(ii) $\frac{d}{dx}\{x^{-n}J_n(x)\} = -x^{-n}J_{n+1}(x)$

9. (a) Show that

$$e^{2tx-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{\lfloor n} H_n(x)$$

(b) $\int_{-\infty}^{\infty} e^{-x^2} H_n(x)H_m(x)dx = \begin{cases} 0 & , \text{if } m \neq a \\ \sqrt{\pi}\,2^n \lfloor n, & \text{if } m = n \end{cases}$

10. (a) Prove the first shifting formula of Laplace transformation.

(b) Find the Laplace transform of $\sin^3 2t$.

11. (a) Find the inverse transform of any one of the following :

(i) $\frac{S+2}{s^2-4s+13}$

(ii) $\frac{5s+3}{(s-1)(s^2+2s+5)}$

(b) If $L\{f(t)\} = \bar{f}(s)$, then show that

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\{\bar{f}(s)\},$$

where $n = 1, 2, 3 \ldots$

12. Apply the method of Laplace transform to solve any one of the following differential equations.

(a) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$

with $x = 2, \frac{dx}{dt} = 1$ at $t = 0$

(b) $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4t + e^{3t}$

when $y(0) = 1$ and $y'(0) = -1$

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