2021

Time : 3 Hours

Maximum Marks : 90

D-811

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer any six questions.

1. (a) State $\alpha - \delta$ definition of limit of a function. Prove that every differentiable function is continuous.

(b) If $y = (\sin^{-1} x)^2$, Prove that

$$(1 - x^2)y_{n+2} + (2n + 1)xy_{n+1} - n^2y_n = 0$$

2. (a) State and prove Maclaurin's theorem.

(b) If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

3. (a) Find the condition that the line $x\cos\alpha + y\sin\alpha = p$ should touch the curve

$$\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1.$$

(b) Show that is any curve

$$\frac{\text{Sub normal}}{\text{Sub tangent}} = \left(\frac{\text{length of normal}}{\text{length of tangent}}\right)^2$$

4\. (a) Find the radius of curvature in pedal form.

(b) Prove that for a curve given by

$r^2 = a^2 \cos 2\theta$, we have $e = \frac{a^2}{3r}$

5\. Evaluate any two of the following :

(a) $\int \frac{x^2+1}{x(x^2-1)}\, dx$

(b) $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

(c) $\int (\sqrt{\tan x} + \sqrt{\cot x})\, dx$

(d) $\int \frac{dx}{5+3\cos x}$

6\. Evaluate any two of the following :

(a) $\int_0^a \frac{x^4}{\sqrt{a^2-x^2}}\, dx$

(b) $\int_0^{\pi/2} \cos^n x \cos nx\, dx$

(c) $\int_\alpha^\beta \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$

(d) $\int_0^\pi \frac{dx}{a+b\cos x}\ (a > b > 0)$

7\. (a) If $B(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$, Prove that

$B(m, n) = B(n, m)$

(b) Find the area of a loop of the curve

$r^2 = a^2 \cos 2\theta$

8\. (a) Find the perimeter of the loop of the curve

$9ay^2 = (x-2a)(x-5a)^2$

(b) Find the perimeter of the cardioid

$r = a(1-\cos\theta)$

9. Find the surface area of a right circular cone whose semi-vertical angle is α, height h and base is circular of radius a. Also find the volume of the cone.

10. (a) State and prove Cauchy's general principle of convergence for a sequence.

(b) Prove that

$$\lim_{n\to\infty} \frac{1}{n}\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}\right)=0$$

11. (a) State and prove comparison test to examine the convergence of an infinite series of non-negative terms.

(b) Test the convergence of the series

$$\sum_{n=1}^{\infty}\frac{1}{n^p}.$$

12. (a) State and prove Gauss's test for the convergence of an infinite series.

(b) In an absolutely convergent series, show that the series formed by its positive terms alone is convergent and the series formed by its negative terms alone is convergent.

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