HG(2) — M (4)

Sc. & Arts

2019

Time : 3 hours

Full Marks : 90

Pass Marks : 42

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer any **six** questions.*

1. (a) Show that a ring R is without zero divisor if and only if the cancellation laws hold in R.

   (b) Show that every field is an integral domain.

2. Solve any **two** of the following differential equations :

   (a) $(x-y)^2 \dfrac{dy}{dx} = a^2$

   (b) $\dfrac{dy}{dx} = \dfrac{y}{x} + \tan\dfrac{y}{x}$

   (c) $x\dfrac{dy}{dx} + y = y^2 \log x$

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   (d) $(1 + xy)\, y\, dx + (1 - xy)\, x\, dy = 0$

3. Solve any **two** of the following differential equations:

   (a) $p^2 + 2\, py \cot x = y^2$

   (b) $y = 2px + p^2$

   (c) $y = px + \sin^{-1} p$

   (d) Putting $x^2 = u$ and $y^2 = v$, reduce the equation $x^2 (y - xp) = yp^2$ into Clairaut's form and hence solve it, where $p = \dfrac{dy}{dx}$

4. Solve any **two** of the following differential equations :

   (a) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y = \cos 2x$

   (b) $\dfrac{d^2y}{dx^2} + 4y = \sin 3x + e^x + x^2$

   (c) $\dfrac{d^2y}{dx^2} + y = x^3 + e^x + \sin x$

   (d) $\dfrac{d^2y}{dx^2} + a^2 y = \sec ax$

5. (a) Using method of variation of parameters, solve the following differential equation

$$(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (1+x)e^x$$

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(b) Find the orthogonal trajectory of the family of circles $x^2 + y^2 = 2ax$ each of which touches the y-axis at the origin.

6. Solve any **two** of the following differential equations :

(a) $\frac{dx}{dt} + 4x + 3y = t$, $\frac{dy}{dt} + 2x + 5y = e^t$

(b) $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$

(c) $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$

(d) Using Charpit's method find the complete integral of the equation $p^2x + q^2y = z$

7. Prove the following recurrence relations for the Legendre's polynomial $P_n(x)$:

(a) $(2n + 1)xP_n = (n + 1)P_{n+1} + nP_{n-1}$

(b) $nP_n = xP'_n - P'_{n-1}$

8. (a) Show that $e^{2tx - t^2} = \sum_{n=0}^{\infty} \frac{t^2}{\underline{|n}} H_n(x)$.

(b) Show that :

$$\int_{-\infty}^{\infty} e^{-x^2} H_n(x) H_m(x)\,dx = \begin{cases} 0 & , \text{ if } m \neq n \\ \sqrt{\pi}\, 2^n \underline{|n} & , \text{ if } m = n \end{cases}$$

9. (a) If n is a positive integer, show that $J_{-n}(x) = (-1)^n J_n(x)$.

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(b) Prove the following recurrence formula for $J_n(x)$:

$xJ'_n = nJ_n - xJ_{n+1}$

10. (a) Prove the first shifting property of Laplace Transform.

(b) Find the Laplace transforms of $\sin^3 2t$.

11. (a) Find the inverse transform of any **one** of the following :

(i) $\frac{4s + 5}{(s-1)^2(s+2)}$

(ii) $\frac{s^2 - 3s + 4}{s^3}$

(b) State and prove Convolution theorem.

12. Apply the method of Laplace transform to solve any **one** of the following differential equations :

(a) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$ with $x = 2, \frac{dx}{dt} = -1$ at $t = 0$

(b) $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4t + e^{3t}$ when $y(0) = 1$ and $y'(0) = -1$