Mathematics - 4 (Hons.)

Answer any six questions.

1. (a) Show that every finite integral domain is a field.
   (b) Pove the following in a ring R :
   (i) a (–b) = – (ab) = (–a) b
   (ii) (–a) (–b) = ab
   (iii) a (b – c) = ab – ac

2. Solve any two of the following :

(a) $\frac{dy}{dx}=\sin(x+y)$ (b) $(x^2-y^2)\frac{dy}{dx}=2xy$

(c) $\frac{dy}{dx}=\frac{x+2y-3}{2x+y-3}$ (d) $\frac{dy}{dx}+1=e^{x-y}$

3. Solve any two of the following :

(a) $y=(1+p)x+ap^2$ (b) $y=(1+p)x+ap^2$

(c) $p^2-py+x=0$ (d) $y=2px+y^2p^3$, where $p=\frac{dy}{dx}$

4. Solve any two of the following :

(a) $\frac{d^2y}{dx^2}+a^2y=\sin ax$ (b) $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=xe^{2x}$

(c) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x\cos x$ (d) $\frac{d^2y}{dx^2}-y=xe^x\sin x$

5. (a) Find the orthogonal trajectories of the family of cardoid $r=a(1+\cos\theta)$.
   (b) Using the method of variation of parameters solve the differential equation $\frac{d^2y}{dx^2}+n^2y=\sec nx$.

6. Solve any two of the following partial differential equations :

(a) $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$

(b) $(y+z)p+(z+x)q=x+y$

(c) $(x^2-yz)p+(y^2-zx)q=z^2-xy$

(d) $(y^2+z^2-x^2)p-2xyq=-2xz$ where $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$

7. Apply Charpit's method to find the complete integral any one of the following :

(a) $x^2p^2 + y^2q^2 = z^2$

(b) $p^2 + q^2 - 2px - 2qy + 1 = 0$

8. (a) Prove that $P_n(x) = \frac{1}{2^n \lfloor n} \frac{d^n}{dx^n}(x^2 - 1)^n$.

(b) Prove that $\int_{-1}^{1} P_n(x) P_m(x) dx = 0$ if $m \neq n$

9. Prove the following relations for $J_n(x)$:

(a) $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$

(b) $x J'_n(x) = -nJ_n(x) + x J_{n-1}(x)$

10. (a) Find the Laplace transforms of $t^3 e^{-3t}$

(b) Show that : $L(t \sin at) = \frac{2as}{(s^2 + a^2)^2}$

11. (a) Find the inverse transforms of any one of the following :

(i) $\frac{s^2 - 3s + 4}{s^3}$ (ii) $\frac{5s + 3}{(s-1)(s^2 + 2s + 5)}$

(b) If L {f (t)} = $\bar{f}(s)$], then show that L {t^n f (t)} = $(-1)^n$ where n = 1, 2, 3,

....

12. Apply the method of Laplace transform to solve any one of the following differential equation :

(a) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t} \sin t$, where y (0) = 0 and $y'(0) = 1$.

(b) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^{-t}$, where y (0) = 0 and $y'(0) = 1$.

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