Mathematics - 4 (Hons.)

Answer any six questions.

1. (a) Define Integral domain. Give an example which is an integral domain but not a field.

   (b) Show that every field is an integral domain.

2. Solve any two of the following differential equations :

   (a) $(x+y)^2 \dfrac{dy}{dx} = a^2$ (b) $(1+e^{x/y})dx + e^{x/y}\left(1-\dfrac{x}{y}\right)dy = 0$

   (c) $\dfrac{dy}{dx} = x^3y^3 - xy$ (d) $(1+xy)\,ydx = (1-xy)\,xdy = 0$

3. Solve any two of the following differential equations :

   (a) $y = p^2x^4 - Px$ (b) $p^2y + 2px = y$ (c) $y = px - p^2 + p$

   (d) Reduce the equation $(px - y)(x - py) = 2p$ to Clairaut's form by putting $x^2=u$ and $y^2 = v$ and hence find the general solution.

4. Solve any two of the following differential equations :

   (a) $\dfrac{d^2y}{dx^2} + a^2y = \cos ax$ (b) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = e^{2x} + \sin 2x$

   (c) $\dfrac{d^2y}{dx^2} + y = x^3 + e^x \sin x$ (d) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = x\cos x$

5. (a) Using the method of variation of parameters, solve the differential equation :

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x$$

   (b) Find the orthogonal tranjectories of the series of logarithmic spirals $r = a^0$, where a is parameter.

6. Solve any two of the following partial differential equations :

   (a) $(mz - ny)\,p + (nx - lz)\,q = ly - mx$

   (b) $y^2p + x^2q = x^2y^2z^2$

   (c) $x(y^2+z)\,p - y(x^2+z)\,q = z(x^2-y^2)$

   (d) $(y+z)\,p + (z+x)q = x+y$ where $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial x}$.

7. Apply Charpit's method to find the complete integral any one of the following

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differential equations :

(a) $p^2x+q^2y=z$

(b) $2xz-px^2-2qxy+pq=0$

8. Prove the following recurrence relations for the Legendre polynomial $P_n(x)$:

(a) $(n+1)P_{n+1}=(2n+1)xP_n-nP_{n-1}$ (b) $nP_n=xP_n'-P_{n-1}'$

9. (a) If n is a positive integer, then show that $J_{-n}(x)=(-1)^n J_n(x)$.

(b) Prove that :

(i) $J_{-1/2}(x)=\sqrt{\dfrac{2}{\pi x}}$ (ii) $J_{1/2}(x)=\sqrt{\dfrac{2}{\pi x}}\sin x$

10. (a) Show that when n is a positive integer, Jn(x) is the coefficient of z^n in the expansion of exp $\left\{\dfrac{1}{2}\times\left(z-\dfrac{1}{2}\right)\right\}$ in ascending and descending power of 2.

(b) Prove the following recurrence relation for

$$J_n(x).2J_n'(x)=J_{n-1}(x)-J_{n+1}(x)$$

11. (a) State and prove first shifting theorem of Laplace Transform.

(b) Find the Laplace transform of $e^{-3t}(2\cos 5t-3\sin 5t)$

12. (a) Find the inverse transform of any one of the following :

(i) $\dfrac{s+2}{s^2-4sx+13}$ (ii) $\dfrac{4s+5}{(s-1)^2(s+2)}$

(b) State and prove convolution theorem.

13. Apply the method of Laplace transform to solve any one of the following differential equation :

(a) $\dfrac{d^2y}{dx^2}-2\dfrac{dx}{dt}+x=e^t$ with $x=2, \dfrac{dx}{dt}=-1$ at $t=0$

(b) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+5y=e^{-x}\sin x$ when y (0) = 0 and d_y (0) = 1