MATHEMATICS – 4 (Hons.)

Answer any six questions

1. Solve any two of the following differential equations :

(a) $(3x+y-5)\,dy+2(x+y-1)\,dx=0$

(b) $(x+y+1)\,dx=(2x+2y+3)\,dy$

(c) $\frac{dy}{dx}=y\tan x-y^2\sec x$

(d) $x\frac{dy}{dx}+y=x^2-y^2$.

2. Solve any two of the following :

(a) $y=(1+p)x+ap^2$

(b) $(x-a)^2p^2+(x-y)p-y=0$

(c) $(px-y)(x-py)=2p$

(d) $e^{3x}(p-1)+p^3e^2=0$ where $p=\frac{dy}{dx}$

3. Solve any two of the following differential equations.

(a) $\frac{d^2y}{dx^2}+a^2y=\sec ax$

(b) $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=x^2e^{2x}$

(c) $\frac{d^2y}{dx^2}+9y=x^2$

(d) $\frac{d^4y}{dx^4}-2\frac{d^2y}{dx^2}+y=x^2\cos x$

4. (a) Prove that the system of parabolas given by the equation $y^2=4a(x+a)$ is self orthogonal where a is a parameter.

(b) Find the orthogonal trajectories of the family of cardiodes $r=a(1-\cos\theta)$, where a is a parameter.

5. Solve the differential equation $x^2\frac{d^2y}{dx^2}+(x^2+2x)\frac{dy}{dx}+(x+2)y=x^3e^{x}$ by using the method of variation of parameters.

6. Solve the differential equation:

$$x^2\frac{d^2y}{dx^2}-2(x+1)x\frac{dy}{dx}+2(1+x)y=x^3$$

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7. Solve any two of the following partial differential equations:

(a) $x(y-z)p + y(z-x)q = z(x-y)$ (b) $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

(c) $x^2p^2 + y^2q^2 = z^2$ (d) $p^3 + q^3 = 3pqz$ where $P = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$

8. Apply Charpit's method to solve any one of the following partial differential equations :

(a) $p^2 + q^2 - 2px - 2qy + 1 = 0$ (b) $2z + p^2 + qy + 2y^2 = 0$

9. (a) Prove that $p_n(x) = \frac{1}{2^n \lfloor n} \frac{d^n}{dx^n}(x^2-1)^n$

(b) Prove that $\int_{-1}^{1} [p_n(x)]^2 dx = \frac{2}{2n+1}$

10. Prove the following recurrence relations for the Bessel's function J (X).

(a) $\frac{d}{dx}(x^n)_n(x) = x^n J_{n-1}(x)$ (b) $xJ_n(x) = nJ_n(x) - x^n J_{n-1}(x)$

11. Find the Laplace transform of the following functions :

(a) $(\cos h\, at - \cos at)$ (b) t^3e^{-3t}

12. Apply the method of Laplace transforms to solve any one of the following differential equations :

(a) $\frac{d^2y}{dt^2} + t\frac{dy}{dt} = y$, where $y(0) = 0$ and $\frac{dy}{dt} = 1$ at $t = 0$,

(b) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-1}\sin t$, where $y(0) = 0$ and $\frac{dy}{dt} = 1$ at $t = 0$