## MATHEMATICS – 3 (Hons.)

Answer any six questions.

1. (a) State and prove Leibnitz's theorem to find the nth derivative of a product of two functions of x.

(b) If $y = (\sin^{-1} x)^2$. prove that:

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$$

2. (a) State and prove Euler's theorem on homogeneous functions of two independent variables.

(b) If $u = \cos^{-1}\left(\frac{x-y}{x+y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

3. (a) Prove that :

$$\frac{1}{p^2} = u^2 + \left(\frac{du}{d\theta}\right)^2$$

(b) Find the pedal equation of the curve :

$$r^m = a^m \cos m\theta$$

4. (a) Find the radius of curvature for the cartesian curve $y = f(x)$.

(b) Find the radius of curvature at any point of the curve $r = a(1 + \cos\theta)$.

5. Evaluate any two of the following:

(i) $\int \frac{dx}{1+x^3}$ (ii) $\int \frac{dx}{4+\sec x}$ (iii) $\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

6. Evaluate the following :

(a) $\int_0^{x/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}$ (b) $\int_0^{x} x \log \sin x \, dx$

7. (a) Find the area included between the curve $y^2(a - x) = x^3$ and its asymptote. (b) Find the area of the cardioid $r = a(1 + \cos\theta)$.

8. (a) Find the perimeter or the loop of the curve $3ay^2 = x(x-a)^2$

(b) Find the volume of the solid formed by the revolution of the loop of the curve $y^2 = \frac{x^2(a-x)}{a+x}$ about the x-axis.

9. (a) Prove that $B(m,n) \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$

(b) Find the maxima and minima of the function $x^3 + y^3 - 12x - 3y + 20$.

10. (a) Show that every convergent sequence is bounded.

(b) Discuss the convergence of the sequence $(a_n)$ defined by:

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots + \frac{1}{n+n}$$

11. (a) State and prove Raabe's test.

(b) Test the convergence or the series:

$$x + \frac{2^2 x^2}{\underline{|2}} + \frac{3^3 x^3}{\underline{|3}} + \frac{4^4 x^4}{\underline{|4}} + \frac{5^5 x^5}{\underline{|5}} + \ldots\ldots$$

12. (a) Show and prove Cauchy's condensation test.

(b) Show that the series whose nth term is $\frac{1}{n(\log n)^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$.