2019

Time : 3 hours

Full Marks : 100

Pass Marks : 33

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer **eight** questions selecting at least **one** from each Group.*

Group – A

1. Prove that following :

   (a) $(A \cup B)' = A' \cap B'$

   (b) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

2. Define equivalence relation and show that the relation '<' in the set of all integers is not an equivalence relation.

3. (a) Show that the indentity element in a group is unique.

   (b) Show that the four fourth roots of unity namely $1, -1, i, -i$ form a group with respect to multiplication.

4. Define integral domain and show that the set of integers I is an integral domain with respect to usual addition and multiplication.

5. (a) If A be any square matrix, then show that :

   (i) $(A + A)'$ is symmetric

   (ii) $(A - A)'$ is skew-symmetric

   (b) If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, find AB and BA and show that $AB \neq BA$.

6. Find the inverse of the matrix :

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}.$$

7. (a) If V(F) be a vector space and 0 be the zero vector of V, then show that :

   (i) $a.0 = 0 \; \forall \; a \in F$

   (ii) $a.(-\alpha) = -(a\,\alpha) \; \forall \; a \in F, \forall \; \alpha \in V$

(b) Tho necessary and sufficient condition for a non-empty subset W of a vector space V(F) to be a subspace of V is :

$a, b \in F$ and $\alpha \beta \in W \Rightarrow a\alpha + b\beta \in W$

Group – B

8. (a) Show that every convergen sequence is bounded

(b) Prove that :

$$\lim_{n \to \infty} \frac{1}{n}\left(1 + \frac{1}{2} + \ldots + \frac{1}{n}\right) = 0$$

9. (a) State and prove D'Alembert's ratio test for the convergence of an infinite series.

(b) Test the convergence of the series whose general term is $\left[\sqrt{(n^2+1)} - n\right]$.

10. (a) State and prove comparison test

(b) State and prove Leibnitz's test for the convergence of an alternating series.

11. Define continuity and dfffernitiability of a function at a point. Show that a function differentiable at a point is necessarily continuous at that point.

Group – C

12 (a) Define readical axis and obtain the equations of radical axis of two circles.

(b) Show that the radical axis of two circles is perpendicular to the line joining their centres

13. Find the equation of a parabola in its standard form

14. Show that the sum of the focal distances of any point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from foci S and S', is constant and is equal to 2a

15. Find the angle beween two lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2) Also find the condition for the two lines to be parallel or perpendicular .

16. (a) Find the equation of plane in intercept form.

(b) Find the shrotest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ and also find the equation of the shortest distance.