UG(1) — M (Sub/Gen)

Sc. & Arts – New

2018

Time : 3 hours

Full Marks : 100

Pass Marks : 33

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer **eight** questions selecting at least **one** from each Group.*

Group – A

1. (a) For three sets X, Y and Z, prove that :

   (i) $X \times X = Y \times Y$ implies $X = Y$

   (ii) $X \times Y = X \times Z$ and $X \neq \phi$ implies $Y = Z$

   (b) State and prove the fundamental theorem of equivalence relations.

2. Define countable sets and denumerable sets. Prove that the set Q of all rational numbers is denumerable but the R of all real numbers is uncountable.

3. (a) Define a group and show that if G is a group, $a, b \in G$ then $(ab)^{-1} = b^{-1}a^{-1}$.

   (b) Prove that the cube roots of unity form a group under multiplication operation.

4. (a) Introduce the concepts of a ring and an integral domain. Give an example of a ring which is not an integral domain.

   (b) State and prove the following property is an integral domain D :

   $a \cdot b = a \cdot c$ and $a \neq 0$ implies $b = c$, $a, b, c \in D$

5. (a) Define transpose A′ of a square matrix A. Prove that $(AB)' = B'A'$, where A and B are square matrices of the same order $n \times n$.

   (b) If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$, obtain adjoint of A.

6. (a) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}$, a, b, c being real numbers.

   (b) Define an orthogonal matrix and prove that the transpose of an orthogonal matrix is orthogonal.

7. Prove that intersection of any two subspaces of a vector space is a subspace.

Group – B

8. (a) State and prove Cauchy's general principle of convergence of sequence in set R of all real numbers.

   (b) Prove that $\lim_{n\to\infty} x^n = 0$, when $|x| < 1$ and n is a positive integer.

9. (a) Show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \le 1$.

   (b) Test the convergence of the series $\sum_{n=1}^{\infty} u_n$, where $u_n = \frac{\sqrt{n+1}-\sqrt{n}}{n}$, n = 1,2,3,........

10. (a) State and prove Cauchy's root test for the convergence of an infinite series of real numbers.

    (b) Test the convergence of the series, whose nth term is $\left(1+\frac{1}{n}\right)^{-n^2}$, n = 1,2,3,.......

11. (a) Prove that a function f continuous over a closed and bounded interval [a, b], is bounded on [a, b].

    (b) If a real valued function f is defined by

    $f(x) = x^2 \sin\left(\frac{1}{x}\right)$, when $x \neq 0$,

    $= 0$, when $x = 0$

    then prove that f is differentiable at each point x in R.

Group – C

12. (a) Obtain a condition for two circles to intersect orthogonally.

    (b) Find the equation of a circle which passes through the origin and cuts the circles. $x^2 + y^2 - 8y + 12 = 0$ and $x^2 + y^2 - 4x - 6y - 3 = 0$ orthogonally.

13. Define an ellipse and obtain the standard equation of an ellipse in cartesian form.

14. Obtain the conditions that the general equation of second degree in x and y represents an ellipse and a hyperbola respectively.

15 (a) Find the equation of a plane in normal form.

   (b) Find the condition that the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ may lie in the plane $ax + by + cz + d = 0$.

16. Show that the following two lines intersect

$$\frac{x-9}{3} = \frac{y-1}{5} = \frac{z-18}{-7};\ \frac{x-5}{13} = \frac{y-11}{5} = \frac{z+6}{3}.$$