UG(I) — M (Sub/Gen) New –
Sc./Arts.

2016

Time : 3 hours

Full Marks : 100

Pass Marks : 33

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer **eight** questions in **all**, selecting at least **one** question from each Group.*

Group – A

1. Prove that the following :
   (a) $(A \cup B)' = A' \cap B'$
   (b) $A \subseteq B$ and $C \subseteq D \Rightarrow A \times C \subseteq B \times D$

2. State and prove fundamental theorem on equivalence relation.

3. (a) Do the set I of all integers with the binary operation $*$ defined by $a * b = a + b - ab$ form a group ?
   (b) Show that the four fourth roots of unity namely $1, -1, i, -i$ form a group with respect to multiplication.

(Turn over)

4. (a) Show that every field in an integral domain.
   (b) Prove that the set of integers I is an integral domain with respect to addition and multiplication. Is it also a field ?

5. (a) Define transpose of a matrix and show that $(AB)' = B'A'$.
   (b) Show that any square matrix can be expressed uniquely as the sum of a symmetric and skew symmetric matrix.

6. (a) Find the adjoint of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.
   (b) Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$.

7. (a) Show that the necessary and sufficient conditions for a non-empty subset W of a vector space V(F) to be a subspace of V are :
   (i) $\alpha \in W, \beta \in W \Rightarrow \alpha - \beta \in W$
   (ii) $a \in F, \alpha \in W \Rightarrow a\alpha \in W$
   (b) If V(F) be a vector space and 0 be the zero vector of V than show that :
   (i) $a \cdot 0 = 0 \ \forall a \in F$
   (ii) $a \cdot (-\alpha) = -(a\alpha) \ \forall a \in F, \forall \alpha \in V$

Group – B

8. (a) Define limit of a sequence and show that the limit of a sequence is unique.

(b) Show that every convergent sequence is a Cauchy sequence.

9. (a) Show that the series $\sum \frac{1}{n^p}$ is :

(i) Convergent if $p > 1$

(ii) Divergent if $p \leq 1$

(b) Test the convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$.

10. State and prove Raabe's test.

11. (a) Show that a function f that is continuous over a closed interval [a, b] is bounded there.

(b) Show that :

$$f(x) = x\cos\frac{1}{x},\ x \neq 0$$
$$= 0,\ x = 0$$

is continuous but not differentiable at $x = 0$.

Group – C

12. (a) Find a necessary and sufficient condition for the circles :

$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

and $x_2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ to cut each other orthogonally.

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(b) Show that the radical axis of two circles is perpendicular to the line joining their centres.

13. Show that the general equation of the second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a conic.

14. Find the equation of the tangent to the parabola $y^2 = 4ax$ at a point on it.

15. (a) Find the equation of the plane in normal form.

(b) Find the equation of the plane which passes through the point $(2, -3, 4)$ and is parallel to the plane $2x - 5y - 7z = 6$.

16. (a) Find the conditions that the line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

may lie in the plane $ax + by + cz + d = 0$.

(b) Find the shortest distance between the lines and lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

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