HG (1) – M(2) – Sc. & Art

2021

Time : 3 Hours

Maximum Marks : 90

Candidates are required to give their answers in their own words as far as practicable.

Answer any Six questions.

D-402

1. (a) To find the condition that the line $lx + my + n = 0$ may touch the conic $S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

   (b) To prove that the sum of the ordinates of the feet of all the normals drawn from an external point to the parabola is equal to zero.

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2. (a) What conic is represented by $qx^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$? Reduce the equation to standard form and find the latus rectum of the conic.

   (b) Prove that the locus of the foot of the perpendicular from the focus of a parabola on the tangent at any point is the tangent at the vertex.

3. (a) Obtain the polar equation of a conic in the standard form $l/r = 1 + e\cos\theta$.

(b) Find the equation of the tangent to the conic $l/r = 1 + e\cos\theta$ at the point whose vectorial angle is α.

4. (a) If the normal at L, one of the extremities of the latus rectum of the conic $l/r = 1 + e\cos\theta$, meet the curve again at P, show that

$$\frac{l}{SP} = \frac{1+e^2-e^4}{1+3e^2-e^4}.$$

(b) Prove that the locus of the point of contact of the tangents from a given point to a system of confocals is a cubic curve which passes through the given point and through the foci.

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5. (a) To find the condition that the general homogeneous equation $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ of the second degree in x, y, z should represent two planes and to find the angle between them.

(b) Prove that $\frac{a}{y-z} + \frac{b}{z-x} + \frac{c}{x-y} = 0$ represents a pair of planes.

6. (a) Prove that the plane through the point (α, β, γ) and the line $x = py + q = rz + s$ is given by

$$\begin{vmatrix} x & py+q & rz+s \\ \alpha & p\beta+q & r\gamma+s \\ 1 & 1 & 1 \end{vmatrix} = 0$$

(b) Show that the shortest distance between any two opposite edges of the tetrahedron formed by planes $y+z=0$, $z+x=0$, $x+y=0$, $x+y+z=a$ is $\frac{2a}{\sqrt{6}}$.

7. (a) To find the equation of the tangent plane to the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ at any point (x_1, y_1, z_1) on it.

(b) Find the locus of points form which three mutually perpendicular lines can be drawn to intersect the conic $ax^2+by^2=1, z=0$.

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8. (a) To find the condition that the plane $lx+my+nz=p$ should be a tangent plane to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

(b) To prove that six normal can be drawn from an external point to an ellipsoid.

9. (a) If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$. Prove that $\sum\sin 2\alpha=\sum\cos 2\alpha=0$ and $\sum\sin^2\alpha=\sum\cos^2\alpha={}^3/_2$.

(b) Expand $\cos\theta$ in ascending powers of θ.

10. (a) If $x=\log\tan\left(\frac{\pi}{4}+\frac{y}{2}\right)$.

Prove that $y=-i\log\tan\left(\frac{ix}{2}+\frac{\pi}{4}\right)$.

(b) State and prove Gregory's series.

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11. (a) If $\cos^{-1}(u + iv) = \alpha + i\beta$, where u, v, α and β are all real. Prove that $\cos^2\alpha$ and $\cos^2\beta$ are the roots of the equation $x^2 - (1 + u^2 + v^2)x + u^2 = 0$.

(b) Sum to n terms of the series $\cos\theta + \cos 3\theta + \cos 5\theta + ---$ to n terms, and with help of it prove that $1^2 + 3^2 + 5^2 + ---$ to n terms $= \frac{n(2n-1)(2n+1)}{3}$.

12. (a) Sum the series $1 + \frac{\cos 4\theta}{\underline{|4}} + \frac{\cos 8\theta}{\underline{|4}} + ---$ to ∞.

(b) To prove that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ---$ to infinity $= \frac{\pi^2}{6}$.

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