HG (1) — M (1)
Sc. & Arts – New

2018

Time : 3hours

Full Marks : 90

Pass Marks : 28

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any six questions

1. (a) Prove that every equation of dimension n has n roots, and no more.

   (b) Show that the equation $\frac{A^2}{x-a} + \frac{B^2}{x-b} + \frac{C^2}{x-c} + \ldots\ldots + \frac{L^2}{x-\ell} = x - m$, where a, b, c, ..., l are numbers all different from one another, can not have an imaginary root.

2. (a) By a suitable transformation of variable, reduce the cubic equation $a_0 x^3 + 3a_1 x^2 + 3a^2 x + a^3 = 0$ to the form $z^3 + 3Hz + G = 0$ and obtain the relation between the roots of original equation and those of the transformed equation

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   (b) If α, β, γ be the roots of the cubic $x^3 - px^2 + qx - r = 0$, from the equation whose roots are : $\beta\gamma + \frac{1}{\alpha}, \gamma\alpha + \frac{1}{\beta}, \alpha\beta + \frac{1}{\gamma}$.

3. (a) Solve $x^3 + x^2 - 16x + 20 = 0$, by Cardon's method

   (b) Form the equation whose roots are the several values of ρ, where $\rho = \frac{\alpha - \beta}{\alpha - \gamma}$ and α, β, γ are the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$.

4. (a) Define a group. Prove that a, set G with an associative binary operation defined on G is a group if and only if there exists a left identity element and each element of G has a left inverse.

   (b) Prove that a group with 4 of fewer elements is necessarily abelian.

5. (a) State and prove Cayley's theorem on groups.

   (b) Prove that the order of a cyclic group is equal to the order of its generator.

6. (a) Define a normal sub-group of a group. Prove that a sub-group M of a group is normal in G if and only if every left coset of M in G is a right coset of M in G.

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(b) Prove that the intersection of any two normal subgroups of a given group G is a normal subgroup of G.

5 7. (a) Define Hermitian and Skew-Hermitian matrices. Prove that every square matrix can be expressed in one and only one way as a sum of a Hermitian and a skew-Hermitian matrix.

14 (b) If A is any matrix, prove that :

(i) $A.A'$, and $A'.A$ are both symmetric;

(ii) $A.A^H$. $A^H.A$. are both Hermitian where A' and A^H respectively denote the transpose of A and the congujate transpose of A respectively.

8. (a) Find the rank of the matrix $\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$.

2 (b) Prove that a system of non-homogeneous equation AX = B is consistent if and only if rank of the coefficient matrix A equals the rank of the augumented matnx [AB].

9. (a) Find the eigenvalues of the matrix

6 $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.