2021

Time : 3 Hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

Q. No.1 Carries 20 marks and the remaining questions carry 16 marks each.

Answer Six question in all, selection at least one from each Group, in Which Q. No. 1 is compulsory.

1. Choose the correct answer of the following questions:

   (a) A group having order 169 is :

   (i) Non-abelian

   (ii) Abelian

   (iii) Normal

   (iv) Both (ii) and (iii)

   (b) It the mapping f : $C_0 \rightarrow C_0$ defined by $f(z) = z^7$ is an endomorphism of the multiplicative group of non–zero complex numbers, then the kernel of f contains :

   (i) 5 elements

   (ii) 6 elements

   (iii) 7 elements

   (iv) None empty

   (c) If P is the set of all 2×2 matrices of the form :

$$\begin{bmatrix} p+iq & r+is \\ -r+is & P-iq \end{bmatrix}$$

   where P, q, r, s arbitrary real numbers, then P is :

   (i) Skew field

   (ii) Field

   (iii) Only ring

   (iv) None of these

(d) The number of units in the integral domain of Gaussian integers is :

(i) 2

(ii) 3

(iii) 4

(iv) 8

(e) The polynomial $x^2 + x + 4$ over the field F of integers modulo 11 is :

(i) Reducible over F

(ii) Irreducible over F

(iii) Both (i) and (ii)

(iv) None of these

(f) For the syrnmetric Group S_3, which one of the following is true ?

(i) The inner antomorphism corresponding to any two elements are the same.

(ii) The inner antomorphism corresponding to no two elements are the same.

(iii) S_3 is an abelian group

(iv) None of these

(g) The vector space F [x] of polynomials over the field F has :

(i) Two finite bases

(ii) Three finite bases

(iii) No finite bases

(iv) None of these

(h) If L, (S) denotes the set of all linear combinations of the elements of S, then L, (S) is the submodule of a module M, then the necessary and sufficient condition/conditions for a module M to be a direct sum of its two submodules M_1 and M_2 is/are :

(i) $M = M_1 + M_2$

(ii) $M_1 \cap M_2 = \{0\}$

(iii) Both (i) and (ii)

(iv) None of these

Group-A

2. (a) Define conjugate of an element in a group. Also show that the relation of conjugacy in a group is an equivalence relation.

   (b) Prove that every quotient group of a cyclic group is cycle and the converse is not true.

3. (a) Prove that the Centre Z(G) of a group is a normal subgroup of G.

   (b) If G is a group such that $\frac{G}{Z(G)}$ is cyclic, where Z(G) is the Centre of G, then prove that G is abelian.

4. (a) Prove that the set of all endomorphism of a group forms a group with respect to composite of functions as the composition.

   (b) Prove that for an abelian group the only endomorphism is the identity mapping whereas for non-abelian group there exists an

endomorphism of the group which is different form the identity.

5. (a) State and prove Sylow first theorem.

Group- B

6. (a) Prove that every ideal in a Euclidean domain is a principal ideal.

   (b) Prom that the set of polynomials F(x) Over a field, F is not a field.

7. (a) If R is a commutative ring with unity and A is an ideal of R, then prove that the ring of residue classes R/A is an integral domain if A is prime ideal.

   (b) Define Ring isomorphism which at least two examples.

8. State and prove unique factorization theorem for a polynomial over a field.

9. (a) What is module? Write down its general properties and prove any two properties.

(b) Show that every ideals in ring R is a module over itself.

Group- C

10. (a) Define Vector space. Prove that the union of two subspaces of a vector space V(F) is not necessarily a subspace of V(F).

(b) If W_1 and W_2 are subspaces of the vector space V then prove that their sum $W_1 + W_2$ is also a vector subspace of V.

11. (a) Prove that every linearly independent subset of a finitely generated vector space V(F) forms part of a basis of V.

(b) Show, whether the given vectors (1,2,1), (2,1,5) and (-1,1,2) form a basis for R^3 or not ?

12. (a) If T: $U \rightarrow V$ is a linear transformation from a vector space U(F) into a vector space V(F), then show that the range R(T) of T is a subspace of V(F).

(b) Prove that the set of all linear transformations of a vector space forms an abelian group under addition.

13. (a) If W be a subspace of a finite dimensional vector space V(F), then prove that

$$\dim\left(\frac{V}{W}\right) = \dim V - \dim W$$

(b) If T is a homomorphism of a vector space U(f) to a vector space V(f), then show that Ker (T) is a subspace of U (F).

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