# 2021

Time : 3 Hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

Q. No.1 carries 20 marks and remaining questions carry 16 marks each.

Figure in the margin indicate full marks.

Answer six in all, selecting at least one from each group in which Q. No.1 is compulsory.

1. Choose the correct answer from the given alternatives :- $8\times2\frac{1}{2}=20$

(a) Which one of the following is wrong ?

(i) If $P_1$ and $P_2$ are two partitions of $[a,b]$ such that $P_1 \subset P_2$, then for a bounded function f are $[a,b]$ $L(P_1,f) \leq L(P_2,f)$.





(ii) The necessary and sufficient condition for a function $f(x)$ over $[a,b]$ is R - itegrable, is that $U(P,f) - L(P,f) < E$, where P is a partition of $[a,b]$

(iii) Every continuous function is R-integrable.

(iv) Every monotonic function is not R-integrable.

(b) The improper integral $\int_0^1 \frac{dx}{\sqrt{1-x}}$ has a point of infinite discontinuity at :-

(i) $x = 0$

(ii) $x = \frac{1}{2}$

(iii) $x = 1$

(iv) None of these

(c) If two infinite series are A and B then their Cauchy-product converges to :-

(i) $A + B$

(ii) $A - B$

(iii) $AB$

(iv) $A/_B$

(d) If $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are both differentiable at $(a,b)$ of the domain of $f(x,y)$, then $\frac{\partial^2 f}{\partial x \partial y}(a,b) = \frac{\partial^2 f}{\partial y \partial x}(a,b)$ is the statement of:

(i) Schawarz's theorem

(ii) Young's theorem

(iii) Necessary condition of differentiability

(iv) Sufficient condition of differentiability

(e) The argument of a complex number $^{z_1}/_{z_2}$ is the angle through which :-

(i) $z_1$ rotates to coincide with $z_2$ in anticlock direction.

(ii) $z_1$ rotates to coincide with $z_2$ in clockwise direction.





(iii) $z_2$ rotates to coincide with $z_1$ in anticlock direction.

(iv) $z_2$ rotates to coincide with $z_1$ in clockwise direction.

(f) A function $f(x)$ is said to be analytic at a point $\alpha$ if there exists a neighbourhood $|z-\alpha| < \delta$ at all points of which

(i) $f(z)$ is continuous

(ii) $f'(z)$ exists

(iii) $f(z)$ is optimal

(iv) None of these

(g) Which one of the following is true ?

(i) Every open sphere in a metric space is a neighbourhood of each of its points.

(ii) Finite union of open sets in open.

(iii) Arbitrary intersection of open sets in open.

(iv) Closure of a set is open.

(h) Which of the following is wrong ?

(i) $(AUB)' = A'UB'$, where $A'$ is the derived set of A.

(ii) $AUA' = \bar{A}, \bar{A}$ is closure of A.

(iii) $\overline{AUB} = \bar{A}U\bar{B}$

(iv) $(A \cap B)^0 = A^0 U B^0$

## Group-A

2. (a) State and prove fundamental theorem of integral calculus.

(b) If $f(x) = x^3$ on $[0,1]$ and $P = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$ is a partition of $[0,1]$ then find $L(P,f)$ & $U(P,S)$.

3. (a) State and prove Direchlet's test in improper integral.

(b) Prove that the improper integral $\int_0^{\pi} \log \sin x \, dx$ is convergent at $x = 0$, hence evaluate it.

4. (a) State and prove Cauchy's theorem on multiplication of series.





(b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{(\cos nx)}{n^p} (p > 0)$.

5. (a) If $f(x)$ is bounded and integrable on $[-\pi, \pi]$ and $a_n, b_n$ are Fourier coefficients, then $\sum_{n=1}^{\infty} (a_n^2 + b_n^2)$ is convergent, prove it.

(b) Find the F-series of $|\sin x|$ in $-\pi \leq x < -\pi$.

6. (a) State and prove sufficient condition for differentiability of $f(x,y)\ at\ (a,b) \epsilon D \subset R^2$.

(b) Show that the function :-

$$f(x,y) = x^2 \sin\frac{1}{x} + y^2 \sin\frac{1}{y} \quad if\ (x,y) \neq (0,0)$$
$$= 0 \quad if\ (x,y) = (0,0)$$

is differentiable at $(0,0)$

## Group-B

7. (a) Find the area of a triangle with vertices $z_1, z_2, z_3$.

(b) Prove that $\left|\frac{z-1}{z+1}\right| =$ constant and avg $\left|\frac{z-1}{z+1}\right|$ constant are orthogonal

8. Find the necessary and sufficient condition for the complex valued function f(x) to be analytic.

9. (a) Show that the relation $\omega = \frac{5-4z}{4z-2}$ transforms the circle $|z| = 1$ into a circle of radius unity in the $\omega$ − plane and find the centre of the circle.

(b) Find the condition for $\omega = f_{(z)}$ to represent a conformal mapping.

## Group-C

10. (a) Define an open set and prove that in a metric space, every open sphere is an open set.

(b) Let $(X, d)$ be a metric space and $x, x', y, y \epsilon X$, then show that :-

$$|d(x,y) - d(x',y')| \leq d(x,x') + d(y,y').$$

11. State and prove cantor's intersection theorem.

12. Define contraction mapping and state and prove Banach fixed point theorem.

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